Magnetic Levitation Control for Active Vibration Control Systems Based on TSK Fuzzy Model
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Abstract:
This report presents a fuzzy modeling and tracking control methodology for an active vibration control systems comprised of a magnetic levitation actuator and passive elements, spring and damper. The methodology combines the merits of fuzzy logic and conventional linear control theory. Here, TSK (Takagi-Sugeno-Kang) fuzzy reasoning algorithm is employed to formulate a magnetic levitation actuator by aggregating a set of linearized local subsystems. To control and stabilize the unstable nonlinear magnetic levitation actuator, associated with mechanically flexible passive elements, a fuzzy feedback controller is designed on the basis of parallel-distributed compensation principle employing fuzzy reasoning and linear feedback theory. Besides, to reconstruct the immeasurable states of the magnetic levitation actuator, a fuzzy observer designed by using parallel distributed observation procedure. The fuzzy control approach shows superiority over the conventional linear control design theory by extending the operating range and as well as improving disturbance rejection capability. To verify the effectiveness of the fuzzy control system, some simulation studies have been reported out for a U-shaped hybrid electromagnet actuated active vibration control system.

Keyword: Active vibration control system, fuzzy control, linear control, TSK fuzzy model, U-shaped hybrid electromagnet,

1. Introduction & Motivation:
Many high precision manufacturing assemblies require vibration and disturbance isolation stages to isolate undesired effects of outer disturbance sources. The combination of the passive type isolators with active type elements can widen the effective operation range of the isolation system.[1-3]
Electromagnetic actuators have some significant merits such as, no friction, no abrasion, low noise and little vibration etc. Furthermore, combination of classical magnetic actuators with permanent magnets not only increases the energy efficiency but also reduces the magnet size and yields a more compact structure.[1]
In maglev-actuated vibration control systems, vibrations are imposed to the levitating magnetic part by generally two distinct sources, direct disturbance force source and base or support displacements. Successful attenuation of undesired effects of these disturbance sources very much depends on applied control policy and control quality of actuator.[3]
Generally, electromagnetic levitation devices are unstable and also show fairly nonlinear force characteristics; hence the control design procedure requires much more care. In practice, the nonlinear force equation of the electromagnetic suspension device is linearized around a specified operating point for small deviations and then this linearized equation is utilized to design a stabilizing controller via linear control design techniques.[1] However, the linear control design inhibits inherently a limited performance, since the controller has a fixed structure, and even for sensible deviations from specified operating point, the system can be easily unstable.
Using the extended form of the TSK fuzzy reasoning algorithm, nonlinear dynamic systems can be approximated by aggregating linear local models for given operating points. By using linear control theory, stabilizing and as well as tracking controllers can
be designed each one of the linear local model on the basis of parallel-distributed compensation principle. This promising control approach can overcome aforementioned pitfalls of linear control approach. Furthermore, this approach is eligible for observer design as well.[2]

In this report, at first, the fundamentals of the maglev-actuated active vibration control system is simply explained and inherited magnetic levitation actuator nonlinearities are illustrated by using electromagnetic force equation. Then, the linear control design techniques are outlined for both controller and observer design. Construction of a TSK fuzzy model for U-shaped electromagnet is described and for the developed fuzzy model, stabilizing and tracking controller design and also disturbance observer designs are stated in logical manner. To obtain the passive elements displacement from accelerometer measurements, fuzzy identification based electromagnetic force calculation principle is expressed. To verify the effectiveness of the proposed control approach, some simulations are carried out by using Matlab & Simulink packages.

2. Fundamentals of Active Vibration Control System

The investigated active vibration control system have two degrees of freedom motion capability and consists of passive elements connecting to base and magnetic levitation actuator for actuation. Fundamental configuration of the system is depicted in Fig.1.

![Figure 1. Principle configuration of the investigated active vibration control system.](image)

Electromagnetic force developed by magnetic levitation actuator can be expressed in analytical form as;

\[
F_e = \frac{B^2}{\mu_0} S = \frac{\mu_0 SN^2}{4} \left( \frac{i + 2H \mu_m / N^2}{g + \mu_m / \mu_m} \right)^2 = k \left( \frac{i + I_m}{g + \mu_m / \mu_m} \right)^2
\]

(1) is linearized around nominal gap length for small excursions to design a stabilizing controller employing linear control theory as following fashion;

\[
m_g = \left( \frac{I_m}{g_0 + \mu_m / \mu_m} \right)^2
\]

where, \(k\) is the permanent magnets, zero power based control algorithm may be applied.[3] However, equalization constraint of negative and positive stiffness values are violated by practical reasons. To overcome this problem, employment of passive elements’ displacement following gap length type control approach would be more rationale technique.[5] The full system dynamics, in state space, will be comprised of 5 state variables current of actuator coil, upper and lower mass displacements & velocities. The function of the actuator in the active vibration control system is to provide virtual damping and negative stiffness. From this point of view, the control problem can be relaxed if the actuator gap clearance is solely controlled to yield desired damping and negative stiffness.

On the other hand, another issue is the sensor selection and arrangement. To solve this we have addressed following sensors combination;

- Current and displacement sensors for electromagnet,
- Upper and lower mass accelerometers for motion sensing.

Passive elements’ displacement following gap length type control approach utilizes state space integral control algorithm and it can be obtained by extending actuator equation via integral term of error between passive elements’ displacement and actuator gap clearance as,[5]

\[
\frac{d}{dt} \begin{bmatrix} x_1 - x_2 \\ i \\ I \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & K_a & 0 \\ 0 & -R & L \end{bmatrix} \begin{bmatrix} x_1 - x_2 \\ i \\ I \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} F_e
\]

(7)

Specifications of the U-type electromagnet used for numerical studies are given in Table 1;

<table>
<thead>
<tr>
<th>m</th>
<th>4.716[kg]</th>
<th>g_a</th>
<th>0.0057[m]</th>
<th>k</th>
<th>1.571×10⁻⁸[Nm²/A²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>L_m</td>
<td>0.003[m]</td>
<td>I_a</td>
<td>15[A]</td>
<td>S</td>
<td>6.25×10⁻⁴[m⁻³]</td>
</tr>
<tr>
<td>N</td>
<td>200[turn]</td>
<td>K_a</td>
<td>6.1635[N/A]</td>
<td>K_e</td>
<td>10573.8[N/m]</td>
</tr>
</tbody>
</table>

Using well-known Newton’s second law; dynamics of the active vibration control system is derived by as;

\[
m_1 \ddot{x}_1 = -F_e - F_d - F_g
\]

(5)

\[
m_2 \ddot{x}_2 = F_e - F_d
\]

(6)

where, \(F_e\), \(F_d\), \(F_g\) and \(F_d\) represent the developed electromagnetic force, external disturbance force, equivalent damper and spring force, respectively.

3. Linear Controller & Observer Design

In this system, since the magnetic levitation actuator includes permanent magnets, zero power based control algorithm may be applied.[3] However, equalization constraint of negative and positive stiffness values are violated by practical reasons. To overcome this problem, employment of passive elements’ displacement following gap length type control approach would be more rationale technique.[5] The full system dynamics, in state space, will be comprised of 5 state variables current of actuator coil, upper and lower mass displacements & velocities. The function of the actuator in the active vibration control system is to provide virtual damping and negative stiffness. From this point of view, the control problem can be relaxed if the actuator gap clearance is solely controlled to yield desired damping and negative stiffness.

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System poles are located at desired places by using a standard pole placement algorithm. When (5) is closely investigated for the availability of all state variables, it is seen that gap clearance velocity and passive elements' displacement are immeasurable, and thereby, they must be reconstructed from the sensor measurements. For gap clearance velocity, the following observer dynamics can be used by introducing the accelerometer measurements as inputs:

\[
\begin{bmatrix}
\dot{q}_3 \\
\dot{q}_4
\end{bmatrix} = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
q_3 - \hat{q}_3 \\
q_4 - \hat{q}_4
\end{bmatrix} + \begin{bmatrix}
l_1 & l_2 & l_3 & l_4
\end{bmatrix} \begin{bmatrix}
\dot{x}_1 - \hat{x}_1 \\
\dot{x}_2 - \hat{x}_2
\end{bmatrix}
\]

\[
\hat{q}_3 = \dot{x}_1 - \dot{\hat{x}}_2
\]

\[
\hat{q}_4 = \dot{x}_1 - \dot{\hat{x}}_2
\]

where \(L\) matrix represents observer gain matrix. As for passive elements’ displacement, the following calculation principle is employed:

\[
x_1 - x_0 = -\frac{m_1 \dot{x}_1 + F_x}{D_x + S_1}
\]

Notice that frequency variable \(s\) and time variables are represented in hybrid notation. Here, developed electromagnetic force by actuator, \(F_x\), is approximated by (4). Furthermore, the direct disturbance, \(F_d\), is analytically determined as:

\[
F_d = F_x - m_2 \ddot{x}_2
\]

Linear control theory employs the linearized magnet parameters, \(K_d\) & \(K_b\) for controller and observer design purposes. In fact, they are nonlinear functions of current and gap clearance of actuator and will show irregularity for different operating points. The utilization of linear control theory restricts both system performance and operating range of the actuator.

4. Fuzzy Controller & Observer Design

Fuzzy logic based inference algorithms can approximate nonlinear functions in arbitrary accuracy and have some advantages over curve fitting based counterparts especially if there are strong couplings among system variables and as well as system includes more than one dimension.[2]

Among the fuzzy reasoning algorithms, TSK fuzzy inference approach gives some convenience for control design, by employing the well-known linear control design principles. In TSK fuzzy reasoning technique, consequent parts of the fuzzy rules are expressed as polynomial function of antecedent parts of the rules. For dynamical systems, rather than employing polynomial equations at the consequent parts of the fuzzy rules, the local linearized system equations are used. The local linear system equations are aggregated by TSK inference algorithm whose final form shapes a fuzzy model resulting in a nonlinear representation of actual plant. Formally, it is described as;

Rule 1: IF \(z_1\) is \(C_1^1\) and \(\cdots\) and \(z_n\) is \(C_n^1\) THEN \(\ddot{x} = A_1 x + B_1 u\)

Rule \(l\): IF \(z_1\) is \(C_1^l\) and \(\cdots\) and \(z_n\) is \(C_n^l\) THEN \(\ddot{x} = A_l x + B_l u\)

Rule M: IF \(z_1\) is \(C_1^M\) and \(\cdots\) and \(z_n\) is \(C_n^M\) THEN \(\ddot{x} = A_M x + B_M u\)

\[
x = \sum_{l=1}^{M} \alpha_l (A_l x + B_l u)
\]

Since the actual system dynamics is constructed from local linear models, for each one of the local linear model, state space pole placement control can be designed easily which will lead so called parallel-distributed compensation. TSK algorithm plays a vital role in combining control actions for among local linear models. Even though in local point of view, the control command to system shows linear control trend, their joint structure with fuzzy reasoning gives interpolative control command action to system in global sense. In this way, the nonlinearity accounted for system parameters are introduced to control design elegantly. Such control design procedure is developed as;

\[
\ddot{u} = \sum_{l=1}^{M} \alpha_l (K_l x)
\]

TSK fuzzy reasoning based control approach can be readily employed to active vibration control systems since the static nonlinearity involved in the actuator can be effectively integrated.
into fuzzy model in terms of current and gap clearance. From this point of view the implementation issue takes following fundamental steps:

- Linearization points of the electromagnetic force equation are defined according to the specified equilibrium current and gap clearance values and then the local linear models are constructed. For each one of the local linear model state feedback gain matrices are calculated by using a standard pole placement technique.
- Membership functions are defined for current and gap clearance on the basis of local linear models’ number.
- Rule base combining the input membership functions to consequent local linear models are build up.

The issue of system states availability still remains unsettled. However, it can be solved effectively in fuzzy manner by introducing parallel-distributed observation principle. As in the case of fuzzy controller design, for each one of the local linear models, an observer design is handled by utilizing linear control theory as:

\[
\dot{x} = A \dot{x} + B u + L (y - \hat{y})
\]

\[
\hat{y} = C \hat{x}
\]

Since the fuzzy reasoning system extracts the nonlinearities in local linear models of the system in interpolative manner, directly the actuator parameters can be identified for these local regions. Their employment in electromagnetic force calculation gives much more precise results. So, the identification range of direct disturbance forces is widened. The displacement of passive elements can be calculated by using following hybrid equation.

\[
q_i(s) = x_i - x_0 = \frac{-m_i \cdot \ddot{x}_i - K_y(\alpha) \cdot q_5 + K_y(\alpha) \cdot q_3}{K + C \cdot s}
\]

Consequently, controller, observer and identification of actuator parameters can be combined into a single structure. The principle block diagram of the fuzzy reasoning system is depicted in Fig 2.

In simulation studies, 9 local linear models have been employed. The linearization points have been decided by taking account to upper and lower bounds of desired gap clearance change and as well as acting disturbance forces. From this bases, following current and gap clearance data pairs are used for linearization:

<table>
<thead>
<tr>
<th>Current/Gap</th>
<th>Current/Gap</th>
<th>Current/Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7.70 / 0.0022</td>
<td>-6.12 / 0.0057</td>
<td>-4.78 / 0.0092</td>
</tr>
<tr>
<td>-2.76 / 0.0022</td>
<td>-0.11 / 0.0057</td>
<td>2.12 / 0.0092</td>
</tr>
<tr>
<td>2.17 / 0.0022</td>
<td>5.89 / 0.0057</td>
<td>9.04 / 0.0092</td>
</tr>
</tbody>
</table>

The center entry of Table 2, (-0.1151, 0.0057), corresponds to desired nominal operating point of electromagnet. The associated magnet parameters have been obtained as:

\[
\begin{align*}
q_1(s) &= x_i - x_0 \\
&= \frac{-m_i \cdot \ddot{x}_i - K_y(\alpha) \cdot q_5 + K_y(\alpha) \cdot q_3}{K + C \cdot s}
\end{align*}
\]

Since, the system is comprised of 9 local models, to bind antecedent variables, current and gap clearance, to derived local linear models, 9 rules have been defined. To transform the real time input variables of the antecedent part to fuzzy ones, following membership functions are utilized, Fig. 4-5.
Furthermore, the actuator parameter identification scheme is also embedded into the fuzzy reasoning system. This procedure can be carried out by integrating following rule tables into total fuzzy inference system.

**Table 3. Rule base for normalized $K_a$**

<table>
<thead>
<tr>
<th>Gap / Current</th>
<th>Negative</th>
<th>Zero</th>
<th>Positive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1.1330</td>
<td>1.6765</td>
<td>2.2200</td>
</tr>
<tr>
<td>Nominal</td>
<td>0.6758</td>
<td>1.0000</td>
<td>1.3241</td>
</tr>
<tr>
<td>Big</td>
<td>0.4815</td>
<td>0.7125</td>
<td>0.9434</td>
</tr>
</tbody>
</table>

**Table 4. Rule base for normalized $K_b$**

<table>
<thead>
<tr>
<th>Gap / Current</th>
<th>Negative</th>
<th>Zero</th>
<th>Positive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1.3783</td>
<td>1.6766</td>
<td>1.9293</td>
</tr>
<tr>
<td>Nominal</td>
<td>0.8221</td>
<td>1.0000</td>
<td>1.1507</td>
</tr>
<tr>
<td>Big</td>
<td>0.5857</td>
<td>0.7125</td>
<td>0.8199</td>
</tr>
</tbody>
</table>

Consequently, the rule base of the total fuzzy reasoning system takes following form:

**Rule 1:** IF Gap is small AND Current is negative THEN Controller Gain Matrix is $K_\text{a}^1$ AND Observer Gain Matrix $L_\text{a}^1$ AND Normalized Value of $K_a$ 1.1130 AND Normalized Value of $K_b$ 1.3783.

**Rule 9:** IF Gap is big AND Current is positive THEN Controller Gain Matrix is $K_\text{a}^9$ AND Observer Gain Matrix $L_\text{a}^9$ AND Normalized Value of $K_a$ 0.9434 AND Normalized Value of $K_b$ 0.8199.

Note that in rule base, $\text{a}^1$ ... $\text{a}^9$ represents local linear dynamics and its association with feedback and observer gain matrix at the linearization points specified in Table 2.

### 5. Simulation Results

To investigate the effectiveness of the outlined control approaches some simulations have been conducted. Firstly, to observe the electromagnetic force identification accuracy of the fuzzy reasoning system, the active vibration control system is excited by a periodically changing stepwise 10 Nt. external disturbance force. The time change of the direct disturbance force, the gap clearance, the coil current and the exact, fuzzy identified, (20), and approximately calculated, (4), electromagnetic force results are given in Fig. 6.a-d.

**Figure 6. Electromagnetic force ($F_e$) identification results.**

As we see from the results exhibited in Fig. 6, the proposed fuzzy identification technique provides more accurate results than approximate calculation; hence it reveals the feasibility of such a technique.

**Figure 7. Applied disturbances.**

To test various properties of the active vibration isolation system based on previously mentioned control concepts, the system is simultaneously excited by two external disturbance sources, $F_d$ & $x_0$. It has been assumed that the direct disturbance force, $F_d$, has low frequency dominant features in the meanwhile the base displacement, $x_0$, shows dominancy in high frequency range. Therefore, in simulations, $F_d$ has been applied to the system as 5 Nt., stepwise, 0.16 Hz periodic signal and $x_0$ is inserted into...
the system 0.001 m. amplitude, sinusoidal, 20 Hz signal. Their time change is illustrated respectively in Fig. 7.a-b.

At first, estimation precision of the fuzzy observer is investigated and the obtained simulation results for gap clearance velocity are given in Fig. 8.

![Image of Fig. 7: Gap clearance velocity.](image)

The designed fuzzy observer quite accurately estimates the immeasurable state gap clearance velocity as seen in Fig. 7. Then, the linear control and fuzzy control approaches are compared for predefined external excitations (Fig. 7) and the obtained simulation results are illustrated in Fig. 8.a-c.

Fig. 8 reveals the supremacy of fuzzy control over linear control approach. Furthermore, to test the robustness of the investigated control concepts, the disturbance values are doubled and applied to the system. It has been observed that the proposed fuzzy control approach preserves its stability while the linear control approach falls down.

6. Conclusions

In this report, an active vibration control system comprised of magnetic levitation actuator and passive elements has been introduced and to solve vibration isolation issue, passive elements’ displacement following gap length control policy has been proposed. Furthermore, linear control design principles have been outlined and to cure the pitfalls of linear control approach alternatively fuzzy controller, observer design procedures have been proposed and described. Finally, through the simulations the effectiveness and feasibility of the proposed fuzzy approach have been investigated and also the results have been reported out. As future work, we are planning to deal with stability issues of fuzzy control system and also experimentally verify the simulation results on an experimental test bench.

![Image of Fig. 8: Time response of gap clearance, current and upper mass displacement for linear and fuzzy control approaches.](image)

References


