Engineering Significance of Functions of Bi-Articular Muscles in Motion Controls of Robotic Arms and Legs

---What is advantageous for engineers?---

Takafumi Koseki, Hiroyuki Fukusho, and Takahiro Sugimoto
Department of Electric Engineering and Information Systems, School of Engineering, The University of Tokyo
takafumikoseki@ieee.org

Abstract — This paper explains substantial advantage of thinking of biological muscle structure in a motion control of a humanoid robot limb. There are differences between biological muscle structure and present humanoid robots. We have two joint simultaneous drive by bi-articular muscles. In this paper, characteristics and advantages of the bi-articular simultaneous drive on static condition have been mathematically calculated. Furthermore, advantages of such structure and actuation method in dynamic motion will be discussed, assuming specific conditions of our bodies as a consequence of our evolution.

Keywords — motion control, bi-articular muscle, robot, robotic control, bio-mechanics

I. INTRODUCTION

Technologies for humanoid robots are advancing these days. Improvement of motion control of a robot limb is important for applications of robots to human environment. Conventional motion control technology of a humanoid robot like, e.g., ASHIMO of HONDA Co. Ltd. is based on relatively complicated reversed kinematic calculations, which result in heavy computational load. Biomechanic motion control is inherently different from such existing artificial robots. Researchers in biology and medical science recognize the difference and significant role of a bi-articular muscle pair which does not exist in artificial mechanical robots[1][2].

Figure 1 shows muscle and joint components of a human arm. One sees two pairs of mono-articular muscles which connect between two links around a joint. In addition to them, we have a pair of bi-articular muscles, e3 and f3, as illustrated in Figure 1. The bi-articular muscles connect between two links beyond an intermediate link and two adjacent joints, and the bi-articular muscles generate the contractive force to those joints simultaneously. All quadruped and biped animals have the bi-articular muscles, and the authors of [2] write that standard motions using the bi-articular muscles are realized without any complicated dynamic calculations.

However, such a bi-articular actuator is redundant, and may be too heavy and expensive as an engineering actuator in an artificial robotic system. The redundancy has been justified in the evolution of life, since the motion availability even in an injured case is substantially significant for survival of an animal. However in industrial applications, a certain advantage improving motion performance is furthermore requested to justifying the third redundant actuator. This paper is trying to explain the "substantial advantage of imitating structure and motion controls of an existing animal" in engineering artificial robots.

For rational reasoning, we assume that the bi-articular muscle may be advantageous in special structural conditions and under specific motions observed as a consequence of human evolution as explained in the following sections.

II. STATIC TIP FORCE OF AN ARM FROM BI-ARTIVULAR SIMULTANEOUS DRIVE

A. Force-direction control using bi-articular muscles

A benefit of a control of mono-articular and bi-articular simultaneous drives will be mathematically explained in this section. Kumamoto etal., have measured a response of each muscle using electromiogram (EMG) on a static condition in order to know the function of the muscles. They measured the static muscle function by the following way [1][2]. An arm of a man fixed at a specific position as illustrated in the left half in Figure 1. Force is generated from the wrist point toward six-directions while the wrist position is fixed. Six-directions mean positive and negative ones.

Fig. 1. Force direction using EMG (Electromiogram) on static condition.
parallel to the line connecting the shoulder joint and the wrist, parallel to the line connecting the shoulder and the elbow joints, and to the line connecting the elbow joint and the wrist, respectively. Actuating stimuli to each muscle measured by EMG correspond to the six directions as shown in right half in Figure 1. D1, D2, ..., D6 are edge points of the main six directions, and a vector to a point of hexagonal shape from the wrist shows the direction and the amplitude of the force generated by the human arm. The amplitude of the command to each muscle is controlled in the straightforward way as illustrated in Figure 1 [1][2] in static force production.

B. Control of forces of bi-articular muscles in the static conditions

Relationship of joint torques corresponding to individual drive for each joint and bi-articular simultaneous drive, and the resultant output force on the tip point shall be mathematically described in this section. Figure 2 shows analysis model of a two-link arm. The original point is set to the shoulder point. The coordinate on the tip point x is described as follows.

\[ \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{bmatrix} \]

(1)

Where, \( l_1, l_2 \) : Length of each link (m), \( \theta_1, \theta_2 \) : Angle of each joint (rad)

\[ J(\theta) = \begin{bmatrix} l_1 \sin \theta_1 + l_2 \sin(\theta_1 + \theta_2) \\ l_1 \cos \theta_1 + l_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \]

\[ J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \]

(2)

\[ \tau = J^T(\theta)F \]

(3)

\[ \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \]

(4)

\( T_1 \) and \( T_2 \) are rotational torques by mono-articular muscles at shoulder and elbow points respectively, and \( T_3 \) means simultaneous rotational torques at the two joints by bi-articular muscles, to be superposed to the previous two torques. In (4), the torque \( T_3 \) has already been defined from previous researches [3]. When rotational radius of each joint is assumed identical, (3) represents the simplified coordinates-transformation to the joint torques from these actuator torques [4]. From (3) and (4), the force at the tip point \( F \) of the two-link robot model can be formulated using the inverse matrix of the Jacobian matrix. We have defined one special case of that the length of each link is same value (\( l_1 = l_2 = l \)). The force \( F \) at the tip point here is written as follows.

\[ \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \frac{1}{lS_2} \begin{bmatrix} lC_{12} - l(C_1 + C_{12}) \\ lS_{12} - l(S_1 + S_{12}) \end{bmatrix} \begin{bmatrix} T_1 + T_3 \\ T_2 + T_3 \end{bmatrix} \]

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From (5), it is obvious that condition of the same link lengths is important to simplify the mathematical representation of the force \( F \).

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\[ F_x = \frac{\cos(\theta_1 + \theta_2)}{l \sin \theta_2} (T_1 + T_2) \]  
\[ F_y = \frac{\sin(\theta_1 + \theta_2)}{l \sin \theta_2} (T_1 - 2T_2 - T_3) \]  
\[ T_1 = -T, \ T_2 = +T, \ T_3 = +T \]  
\[ F_x = 0 \]  
\[ F_y = -\frac{2}{l \cos \theta_1} T \]  
\[ \text{(10-A)} \]
\[ \text{(10-B)} \]

(10-A) means that the lateral tip force, which may cause undesirable slips at the tip, is never produced in this framework. Furthermore, the constant output tip force can be easily produced by just giving single torque command simply proportional to \( \cos \theta_1 \).

III. DIFFICULTY IN A DYNAMIC MOTION CONTROL

A. Motion equations for two-link robot arm system

Dynamic behaviour of the straight motion of the two-link arm model will be furthermore discussed in this section. Lagrange equation of motion has often been used in conventional robotic controls. It is, therefore, also used in this section. Calculation of joint torques based on the Lagrange equation of motion is executed by a conventional way[5], therefore only especially important equations will be explained here. Analysis model is illustrated in Figure 3. Variables and constants of the model is shown in TABLE I. The length of each link is defined identical to section II. In addition, it is assumed that radius of rotation of each joint: \( r_i \), the mass of each link: \( m_i \), and length between joint \( i \) and centre of gravity of link \( i \)\( l_i \) respectively.

The shape of each link is assumed as cylinder solid here. Therefore, moment of inertia of each link are described as follows;

\[ l_1 = l_2 = m \left( r_1^2 + \frac{r_2^2}{4} \right) \]  
\[ \tau = M(\theta)\ddot{\theta} + h(\dot{\theta}, \ddot{\theta}) + g(\theta) \]  

\[ M(\theta) \] means an inertia matrix, \( h(\dot{\theta}, \ddot{\theta}) \) means summation of a centrifugal and Coriolis forces, and \( g(\theta) \) means gravity term in the equation. Details of terms in (11) are general ones and it is for instance written in [4]. The inertia matrix \( M(\theta) \) is described simply as follows due to the two-link model. Hence, angular acceleration of shoulder and elbow joint \( \ddot{\theta} \) can be also calculated by

\[ M(\theta) = \begin{bmatrix}
M_{11} & M_{12} \\
M_{12} & M_{22}
\end{bmatrix} \]  

B. Characteristic of an acceleration at the tip point

As same as the angular acceleration, tip acceleration of the arm can be expressed in this section. The two-dimensional tip acceleration in Figure 3 is described using angular acceleration of joint and time-derivative of the tip position.

\[ \ddot{x} = J(\theta)M^{-1}(\theta)(\tau - h(\theta, \dot{\theta}) - g(\theta)) + J(\theta)\ddot{\theta} \]  

By applying the assumption of the uniformity of two links, the compensation torques for inertial and gravitational terms in (12) are derived as follows.

\[ \tau_{\text{comp}} = ml^2 \dot{\theta}_1 + mgl \cos \theta_1 \]
\[ \tau_{\text{comp}} = -\left[ \left( 1 + ml^2 \left( \cos^2 \theta_1 - \frac{1}{4} \right) \right) \dot{\theta}_1 + ml^2 \dot{\theta}^2 \sin \theta_1 \cos \theta_1 - \frac{1}{2} mgl \cos \theta_1 \right] \]  

\[ \text{(15)} \]

C. Characteristics of straight motion on the z-axis

It has been discussed in Section II that undesirable lateral tip force \( F_x \) can be controlled as zero when the tip is kept on the y-axis and actuator torques fulfill the relationship in (9).

IV. SUBSTANTIAL SIMPLIFICATION OF THE COMPLICATED INERTIA FORCE NONLINEARITY

A. General joint torques as superposition of inertial effect and the external tip force

Control of each joint torque during a dynamic motion as illustrated in Figure 4 will be concretely discussed by calculating joint torques as superposition of inertia, gravity and tip-force effects. In general, d’Alembert’s principle is applied for the dynamic mathematical formulation.

First, d’Alembert’s principle is simply explained. Summation of the differences between the forces acting on a system and the time derivatives of the momenta of the system itself along any virtual displacement consistent with the constraints of the system, is zero.

\[ F - m \frac{d^2r}{dt^2} = 0 \]  

From this principle, if the feedforward torque compensation of the inertial and gravitational forces in (12) is successful, one can control the tip force based on the simplified relationship between torque \( T \) and force \( F \) in (10-A) and (10-B).
C. How can be simplified the dynamic terms: nonlinear tables of inertial terms

By introducing the simple torque command in (9), a stretching two-link arm can be reduced to the dynamics of one of an ideal virtual linear motor without any lateral slipping force at its end by compensation of the inertial and gravitational effects based on (15). The equations (15) seem still relatively complicated and nonlinear, but the compensation torques solely depend on $\theta_1$. $\tau_{\text{comp}}$ has just a term proportional to $\dot{\theta}_1$ and $\tau_{\text{2comp}}$ is a superposition of the terms proportional to $\dot{\theta}_1$ and $\ddot{\theta}_2$, in addition to the gravitational terms proportional to $F_y$ in (10-B).

This model reduction to a virtual one-directional virtual linear motor is also useful for flexible shock absorption. By applying virtual stiffness and damping controls to this reduced virtual linear motor, arm/leg motion with viscosity and elasticity can be realized in a straightforward way. The simplified realization of the flexible shock absorption with the guarantee of no lateral tip slips from (10-A) is a substantial engineering advantage of emulating actual arm/leg structure and actuation.

Under these conditions, the production of stretching output tip force without any lateral slips is guaranteed. Also the dependency of the torque on the arm angle is substantially simplified for constant tip force.

When the inertial effects of the arms/legs are not negligible, one must consider the active compensation of the effects. Although the inertial terms are quite complicated and nonlinear in general, the mathematical forms are also drastically simplified under the assumption above. Therefore, the implementation of feedforward compensation of the nonlinear effects of the own inertia and the gravity can be relatively easy, and this compensation availability supports the engineering advantage of the simpleness of the external tip force production summarized above for dynamic robotic missions, e.g., pushing, walking and jumping in a case of a leg illustrated in Figure 5.

We are trying numerical and experimental verifications of the advantages of simple rules of bi-articular simultaneous actuation as well as the simplified nonlinear inertial force compensation by using a test robotic arm in Figure 6.

V. CONCLUSION

In this paper, differences between biological arm/legs and artificial humanoid robots have been discussed by focusing the engineering advantage of learning from present animals. Concretely, advantage of bi-articular muscles and their simultaneous actuation to two adjacent joints were analysed.

Although the advantage of the simultaneous actuation is not obvious in general framework, the advantage is straightforwardly understood in static actuation of straightly stretching force output under the following assumptions as a consequence of our biological evolution:

(1) the lengths of two links are identical, and
(2) the output torque of three actuators follows the following patterns identical to the measured results of human EMG: $T_1=T$, $T_2=-T$ and $T_3=-T$.

REFERENCES